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EFFECTS OF THERMAL CONVECTION CURRENTS ON FORMATION OF ICE

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Abstract—A Mach–Zehnder interferometer was used to study the role of convection currents in the formation of ice. With this experimental set-up, the isotherms are obtained throughout the liquid, and from these the convection pattern can be deduced. The critical Rayleigh number for freezing from below is about 480; when this number is exceeded there is coupling between the water–ice interface and the thermal convection currents. The dendrites that initially form in the metastable, supercooled portion of the water extend to the 0°C isotherm.

NOMENCLATURE

- d, depth of unstable layer;
- g, gravity;
- K, thermometric conductivity;
- R, Rayleigh number;
- ΔT, temperature difference;
- X, width of test section;
- α, coefficient of volume expansion;
- $\Delta\delta$, change in index of refraction;
- λ, wavelength of laser light;
- ν, kinematic viscosity.

INTRODUCTION

THE PURPOSE of this study is to gain insight into the process of solidification. Attention is focused on the interaction between the thermal convection currents in the liquid and the formation of the solid phase at the instant solidification begins. Water was chosen as the working fluid because of its importance in atmospheric and terrestrial problems. Freezing of stratified layers of water occurs on various scales—ranging from large lakes to small crevices in rocks. In addition, an understanding of the general process of solidification may result in improvements in castings, purification of materials, and crystal growth. The experimental techniques to be described in this study can be applied to other fluids—the only restriction being that the fluid is transparent to the visible (or near visible) portion of the spectrum.

Previous studies have shown that thermal convection currents play an important role in the freezing of water [1–3]. However in these studies there is no detailed information concerning the pattern of the convection currents in the vicinity of the solid–liquid interface. Thermocouples (or thermistors) were used to determine temperatures at a few isolated points in the flow field. From these measurements, it was difficult if not impossible, to deduce the convection pattern. Townsend [2] froze water from below in a rectangular tank having a cross section of 11·8 × 11·8 in. and a depth of 5·9 in. He found that the largest temperature fluctuations occur in a shallow layer just within the region of stable stratification. He attributes this to internal waves that are excited by impacts of columns of unstable fluid on the base of the stable region. Boger and Westwater [3] froze water from both above and below in a rectangular tank having a cross-section of 0·5 × 0·5 in. and a depth of 2 in. They reported the onset of natural convection to occur at a Rayleigh number of about 1700. In their experiments, they found the interfacial velocity is affected by convection currents; however no detailed study of the convection pattern was made.

In the present study, Bénard cells appear in the form of rolls which are clearly seen on interferograms. Each fringe of the interferogram corresponds to an isotherm. Small particles of dye are introduced into the system and the streak lines show the convection pattern. Since there exists a relation between the convection currents and isotherms, one obtains not only the temperature distribution but also convection pattern from the interferograms. At the onset of freezing it will be seen that the water–ice interface conforms with the convection pattern. Strong coupling will be seen between the Bénard cells and the ice formation.

APPARATUS AND EXPERIMENTAL PROCEDURE

For a fluid with an adverse vertical temperature gradient, the linearized stability theory does not determine the basic structure of the cells—whether hexagonal, rectangular, triangular or two-dimensional rolls. Generally, one thinks of these cells to be hexagonal and three-dimensional, because these appeared in the original work of Bénard [4]. However the experimental work of Koschmeider [5] and Chen and Whitehead [6] indicate the cell pattern depends on various factors, but is generally two-dimensional in the form of rolls. Therefore it was decided that an attempt would be made to use a Mach–Zehnder interferometer to study the convection currents since this instrument has proved to be ideal in two-dimensional heat transfer and fluid flow problems. From interferograms, absolute temperature measurements may be obtained.

Plane, front-surfaced mirrors, beam splitters, and windows used in this system are flat to 1/20 wavelength. The mirrors and beam splitters are 4 in. dia. A 15 mW neon–helium laser is used as a light source for the interferometer. In conjunction with a focusing lens and spatial filter, this provides a point source of monochromatic light. The test section is rectangular having a cross section of $2{\cdot}5 \times 2{\cdot}75$ in. and a depth of $\frac{3}{4}$ in. The top and bottom of the test section are made of brass plates $\frac{3}{8}$ in. thick. The surfaces of these plates which form the test section, are ground to provide smooth surfaces: and thermoelectric cooling modules are mounted to the exterior of these plates. Thus, the plates can be cooled to preselected temperatures by controlling the power input to the thermoelectric modules. The spacing between the two brass plates ($\frac{3}{4}$ in.) is determined by the height of plexiglas spacers that served as ends of the test section. These spacers have troughs machined in them in order to eliminate any pressure changes in the test section due to thermal expansion or contraction of the liquid. The front and back of the test section—through which the laser light passes—are windows. Each of these windows is made of two optical flats (thermopane fashion) with a vacuum between them. These windows are not in direct contact with the upper and lower plates—being separated by a $\frac{1}{8}$ in. thick rubber gasket. This arrangement eliminates heat conduction which would have interfered with the interferograms due to frost deposits on the outside of the windows and convection currents in the surrounding air. The entire optical set-up is mounted on a vibration-isolated table that dampens out vibrations having frequencies greater than 1·5 Hz. A schematic of the test section is shown in Fig. 1.

Interferograms are taken with either a 4×5 in. or 35 mm camera having focal plane shutters. The shutter speeds are 1/125 s and 1/250 s for the 4×5 in. and 35 mm cameras respectively. Front-surfaced mirrors are used to image the center plane of the test section on the film. There were no lenses in the system except at the spatial filter assembly.

Copper–constantan thermocouples are inserted into holes in the brass plates (top and bottom of the test section) $\frac{1}{32}$ in. below the surfaces. These thermocouples are connected to a digital voltmeter that prints the output at time

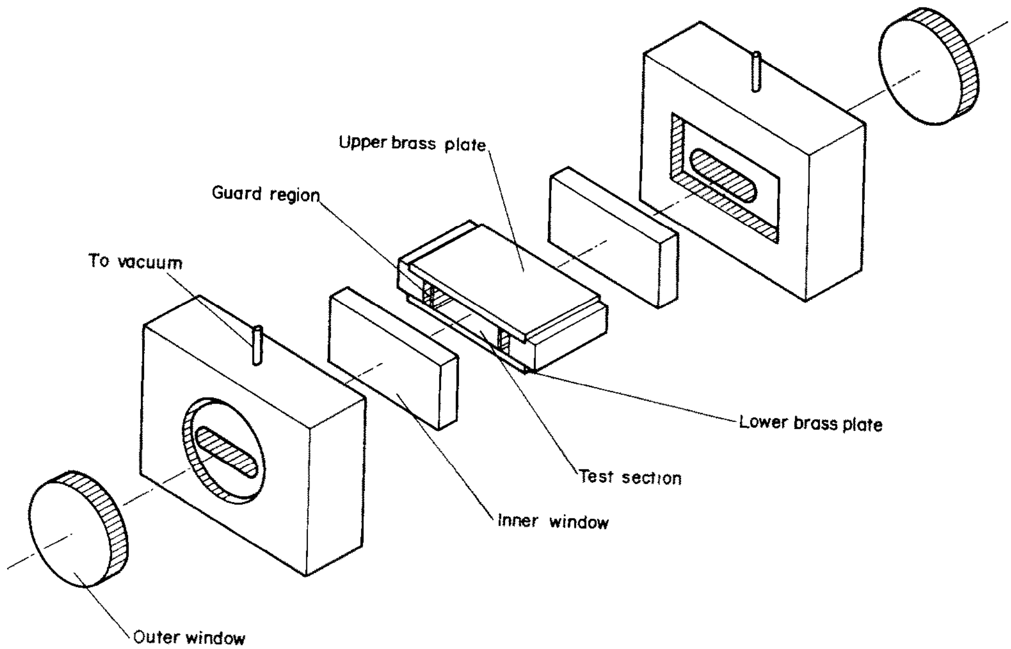

FIG. 1. Schematic drawing (exploded view) of the test section.

intervals of 1·5 s. Because the reference junction temperature is not known precisely, these thermocouple readings are accurate to about 0·5°C. They are not an essential part of the experiment—only indicating when freezing is about to begin so that pictures of the interferograms may be taken. The temperatures in the test section are obtained directly from the interferograms. In some preliminary tests, thermocouples were inserted into the test section to confirm the temperatures obtained from the interferogram by measuring the fringe shifts. These thermocouple readings were recorded on a 10 in. strip chart recorder. Again their accuracy was not better than 0·5°C.

Boiled, distilled water is used to fill the test section—care is taken to eliminate air bubbles within the test section. The test section is reasonably clean, but no special care is taken to have an ultra pure system. This is not necessary, or even desirable, for the experiments in this study. When assembled, the test section is mounted so that its windows surfaces are normal to the laser beam of the interferometer.

With the optics in close alignment and the test section in place, the final adjustments of the interferometer are made. With the water in the test section at a uniform temperature, the instrument can be adjusted for finite fringe spacing or infinite fringe spacing. For this experiment, the infinite fringe spacing is used. When a temperature variation occurs in the test section, each fringe is a contour line of constant optical path. Knowing the optical path (width of test section) and the wavelength of light (6328 Å), the change in index of refraction for a single fringe shift is given by

$$\Delta\delta = \frac{\lambda}{X} \tag{1}$$

where λ is the wavelength of the laser (6328 Å) and X is the width of the test section (7·0 cm)

For this experiment $\Delta\delta$ is 9×10^{-6}. Using a plot of a index of refraction vs. temperature [7] and knowing the temperature at one point in the system, enables one to obtain the temperature at all points. Each fringe is a line of constant temperature.

The optical system is very stable. Once the interferometer is adjusted, it remains in alignment from day to day. However the infinite fringe configuration is checked at the beginning of each set of tests.

EXPERIMENTAL RESULTS

Introduction

Consider the case of a horizontal layer of fluid that has an adverse temperature gradient formed by heating or cooling the lower boundary. By adverse temperature gradient is meant that the heavier fluid is on top and the lighter fluid underneath. This is potentially an unstable situation. If the adverse temperature gradient exceeds a certain value, then Bénard cells are formed. In this experiment, when the lower plate is cooled below 4°C, an adverse temperature gradient appears below the 4°C isotherm. Above the 4°C isotherm the fluid is stable. Freezing from below results in a potentially unstable condition within the freezing zone. When freezing from above, there exists a potentially unstable region if the lower plate temperature is greater than 4°C. But, in this case, the potentially unstable region is outside the freezing zone.

Interpretation of interferograms

Tests were initially performed by cooling the lower surface and watching the fringe pattern develop. Figure 2 is a typical interferogram showing the straight fringes (isotherms) throughout most of the test section. This indicates a stable situation in which no convection currents are present. The band near the bottom plate is a region of constant index of refraction. This occurs at 0·03°C where the index of refraction for water is maximum at 6328 Å. If the optics were perfect, this would be a wide band having no curvature. It is also seen from this interferogram that the top and bottom plates are of uniform temperature since the fringes are flat in the vicinity of the plates.

A series of tests were conducted with thermocouples inserted horizontally in the test section (the thermocouple bead is approximately 0·05 in. dia.). A typical interferogram is shown in Fig. 3. The curvature of the fringes is due to thermal convection currents (Bénard cells). The measured thermocouple temperatures are 4°C and 11°C. A temperature profile was obtained by making fringe count along line AA. Since the lower thermocouple is not on AA, the fringe nearest the lower thermocouple is assumed to be 4·5°C. With the temperature of this fringe assigned, a plot of temperature distribution (solid points) vs. non-dimentional height is shown in Fig. 4. The value of 11°C obtained from the interferogram checks well with the

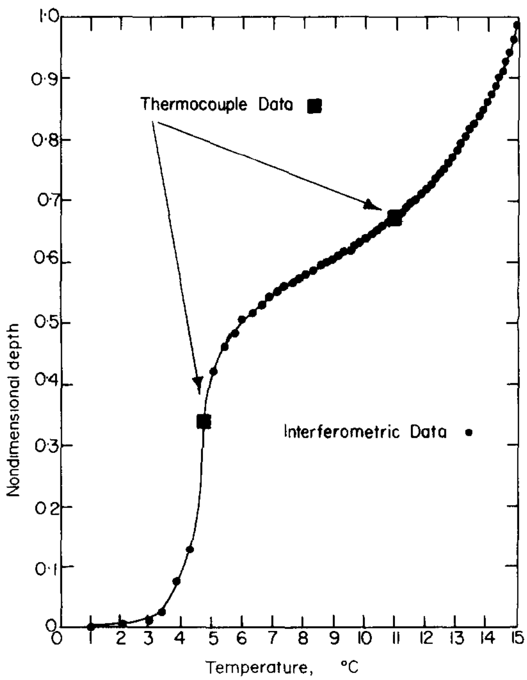

FIG. 4. Temperature distribution along line AA in Fig. 3. Temperatures obtained from interferogram are compared with thermocouple temperatures.

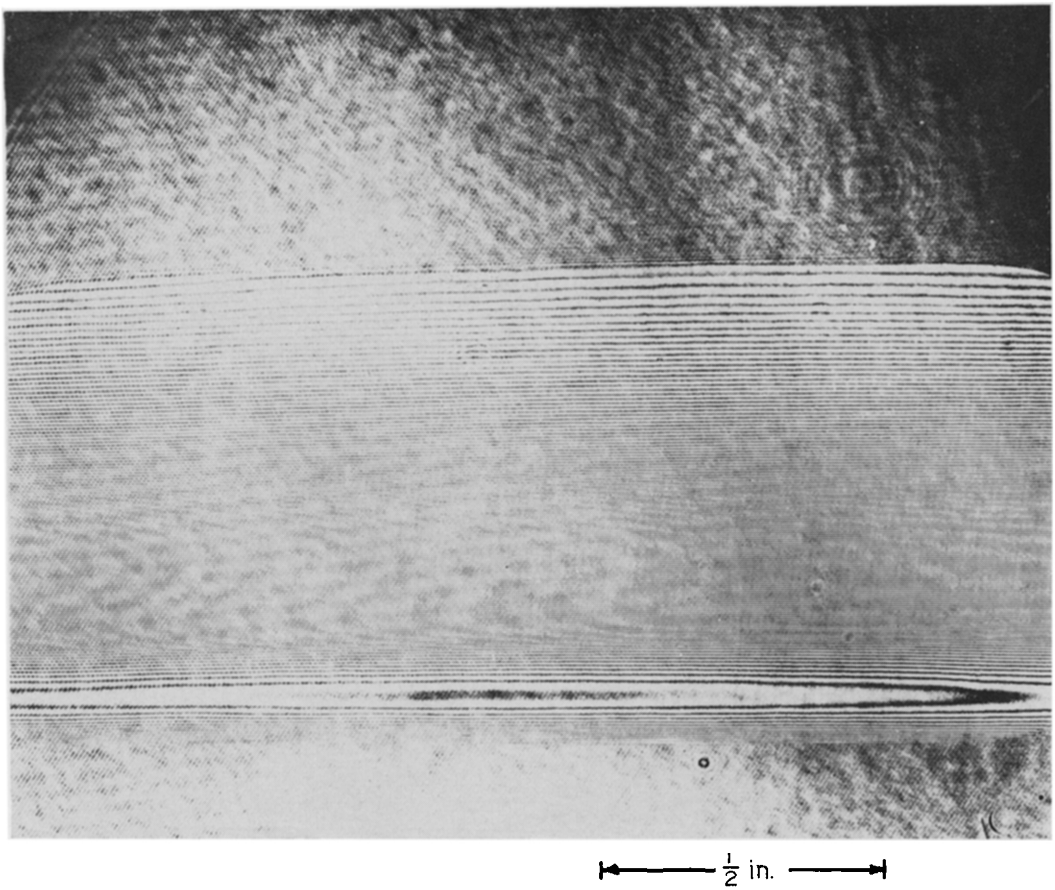

FIG. 2. Interferogram showing straight fringe pattern indicating no convection currents. Bottom plate temperature is −5 °C; top plate temperature is 23½ °C. Temperature at center of wide band near bottom is 0·03 °C.

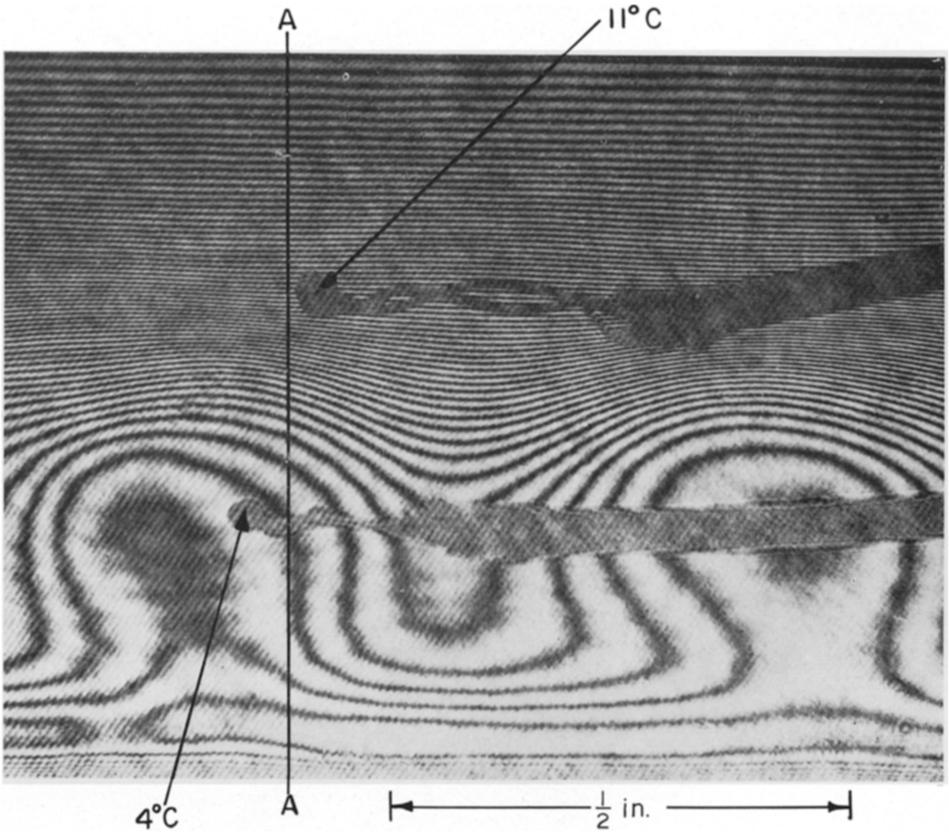

FIG. 3. Interferogram with thermocouples in test section.

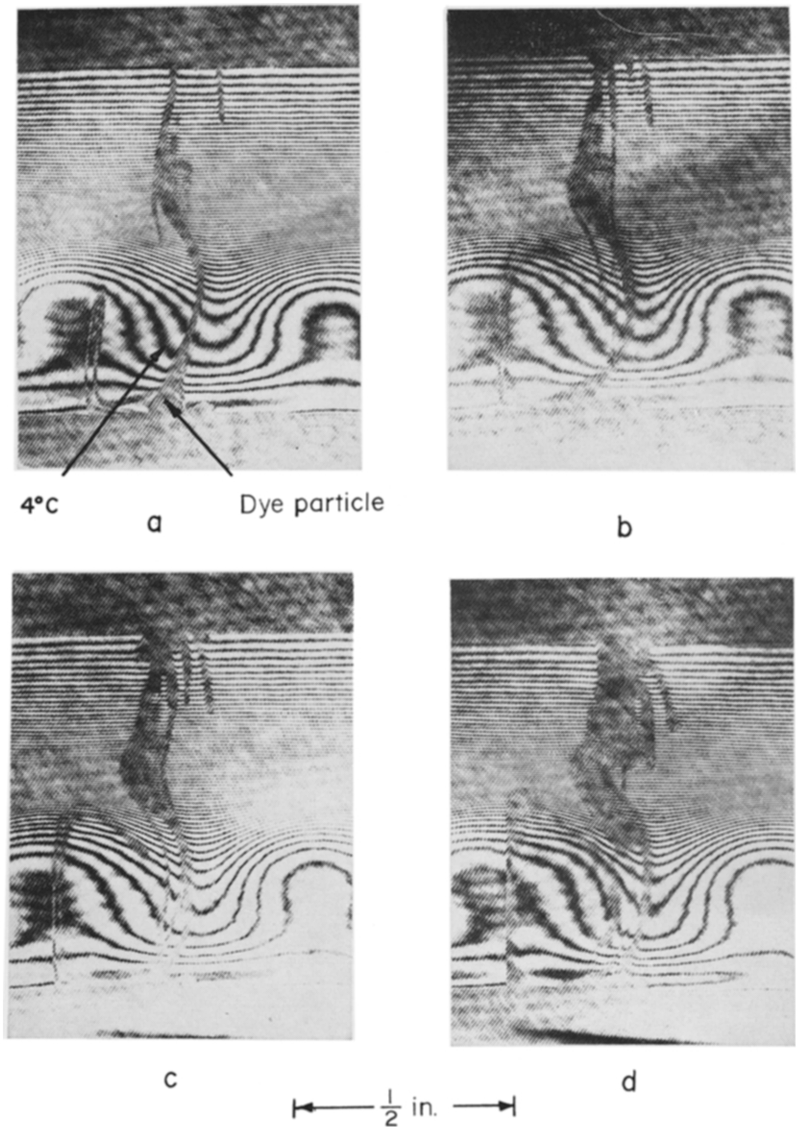

FIG. 5. Interferogram with a dye filament which shows convection currents (streak lines). Time interval between each interferogram is about 2·5 s.

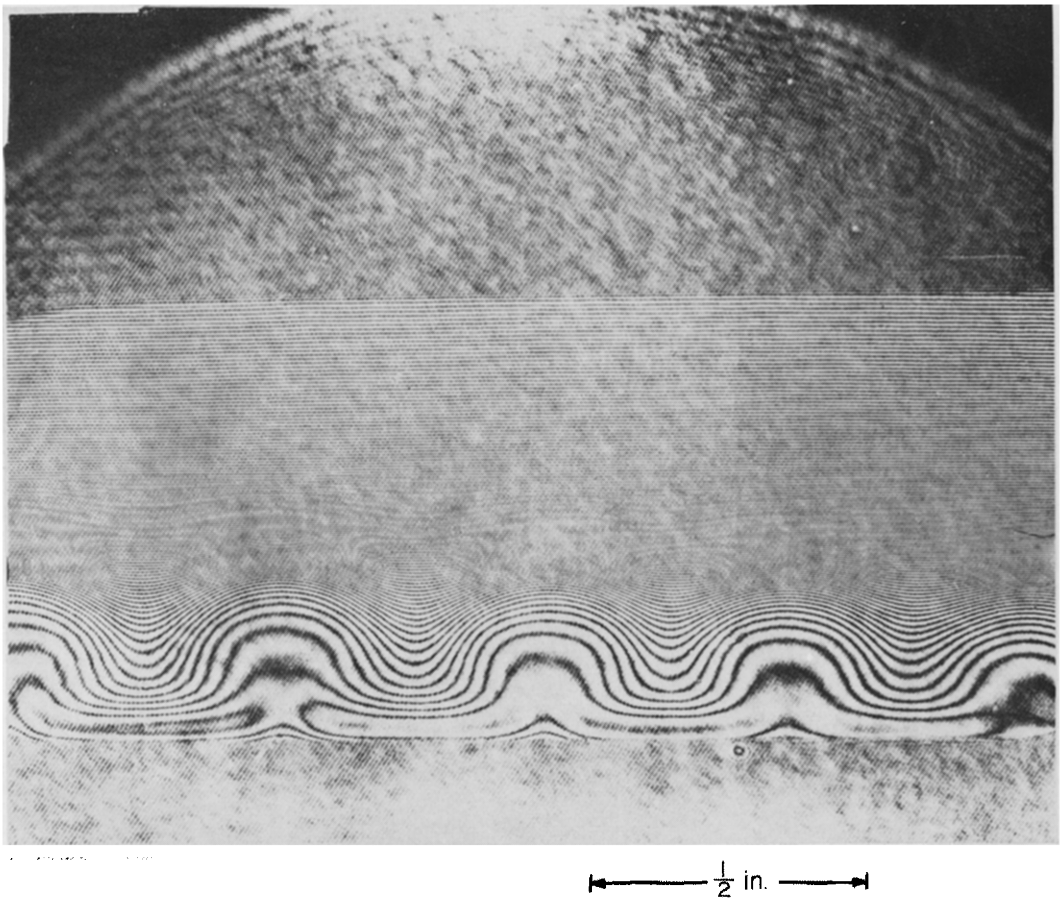

FIG. 7. Interferogram showing the cells that develop after the onset of instability.

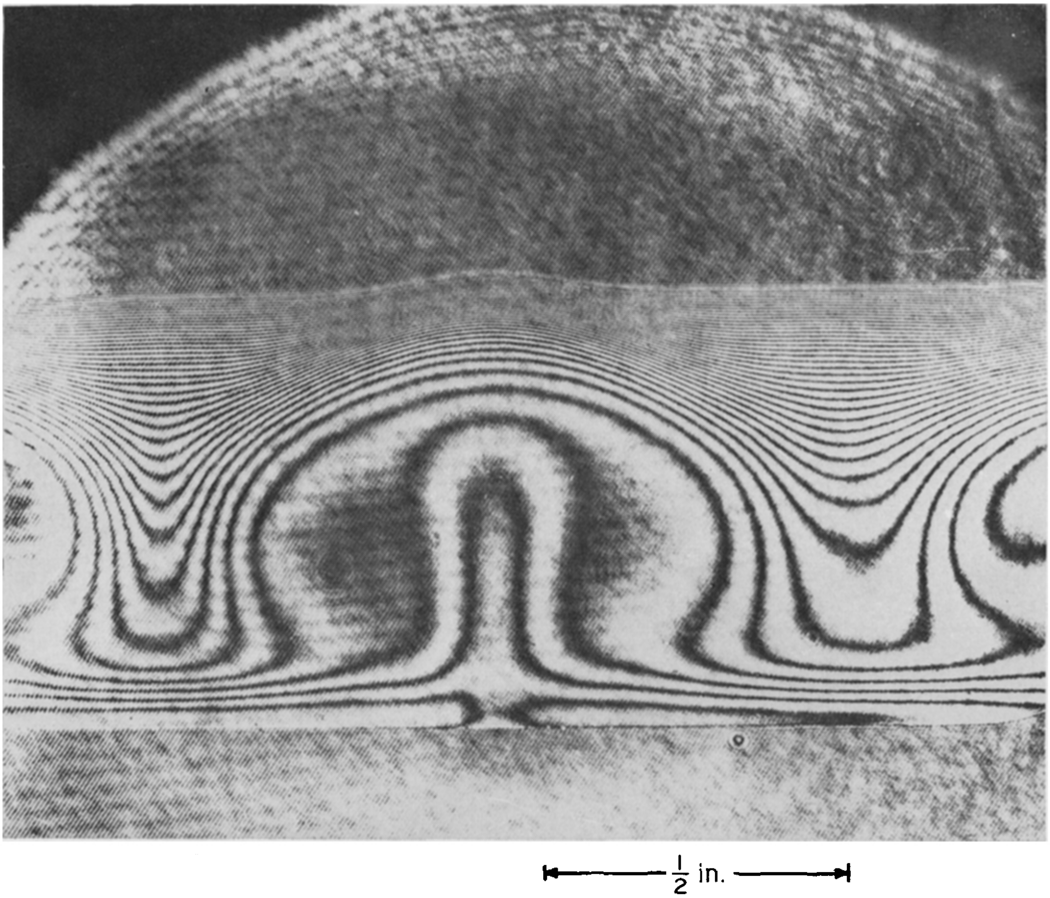

FIG. 8. Interferogram showing large cells that nearly fill the test section.

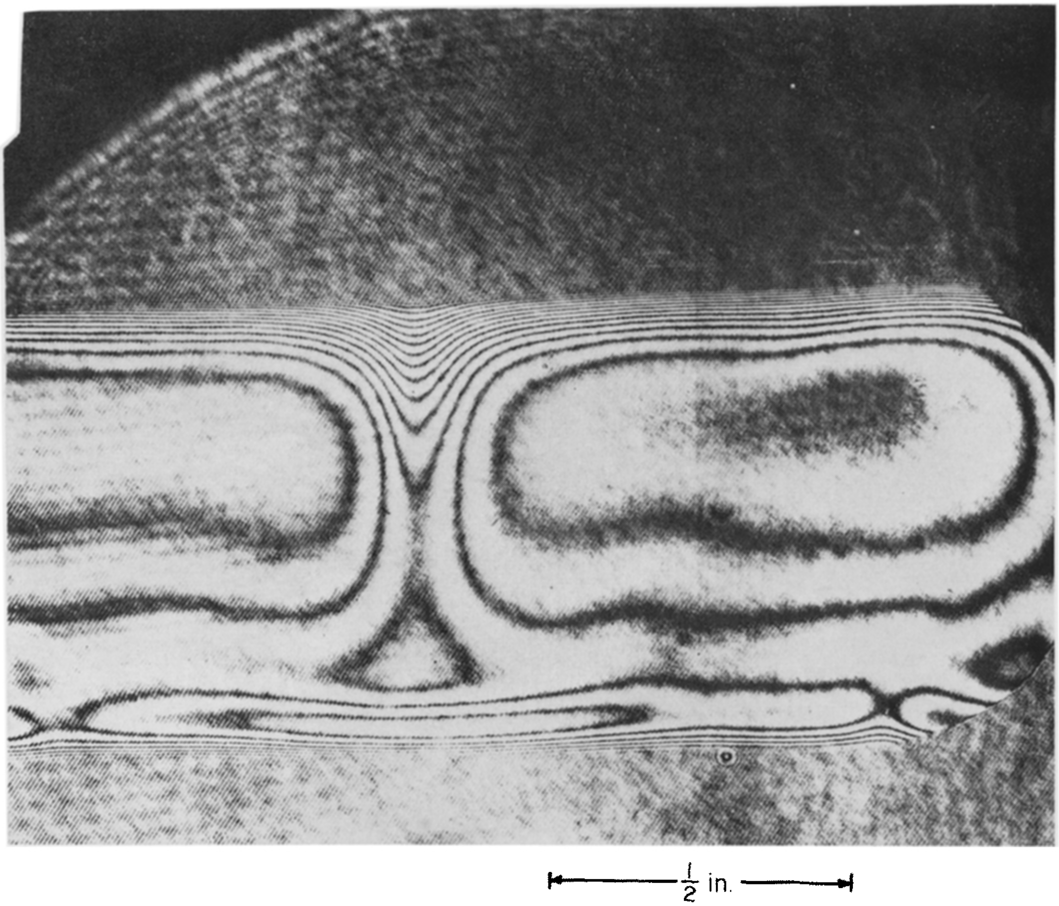

FIG. 9. Interfogram showing large cells that have the opposite circulation pattern to that in Fig. 8.

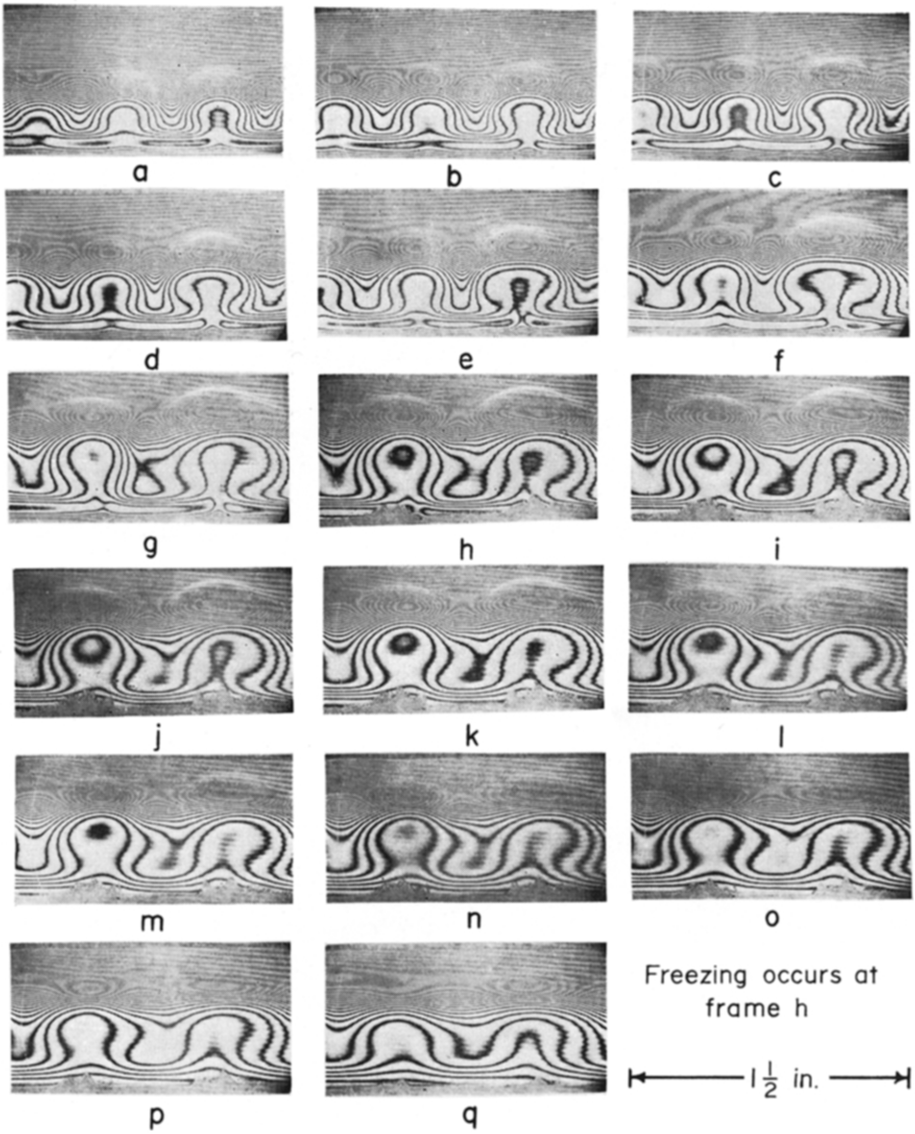

FIG. 10. A sequence of interferograms showing freezing at the bottom. (The secondary fringe pattern seen in the upper half of each picture does not appear on the original photographs. This unavoidable pattern has appeared due to the reproduction process used. On the large interferograms this pattern does not occur.

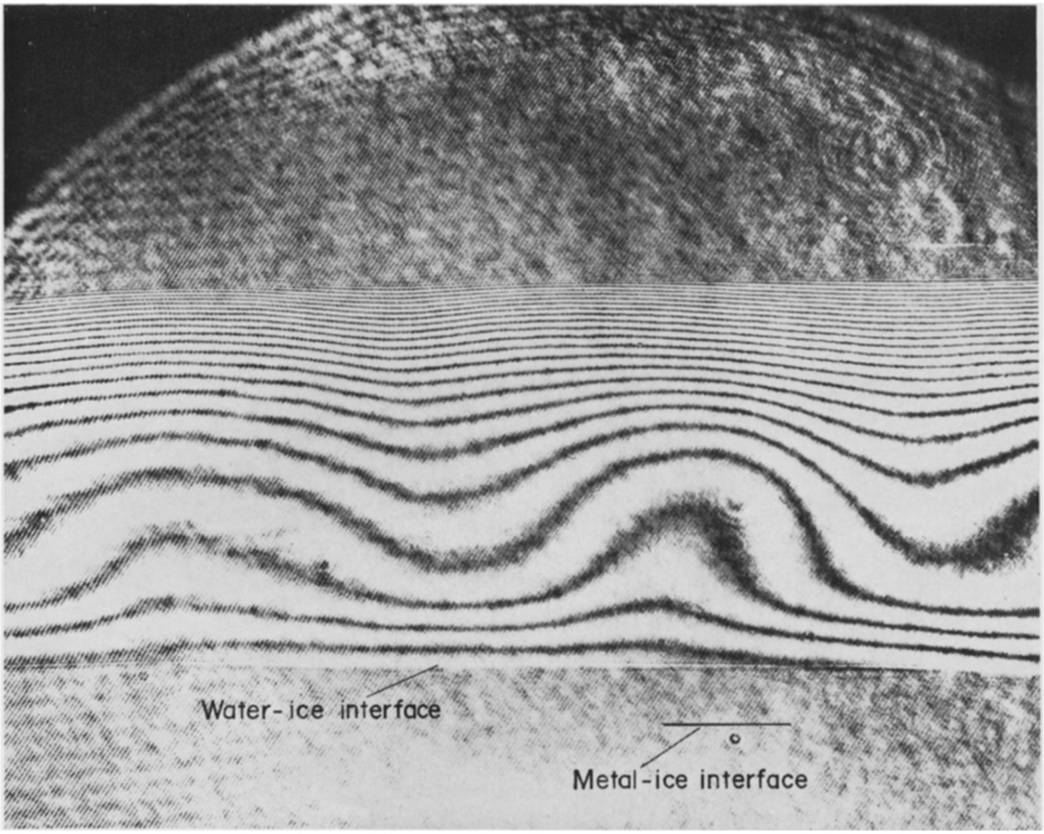

FIG. 11. Interferogram when ice layer is thick.

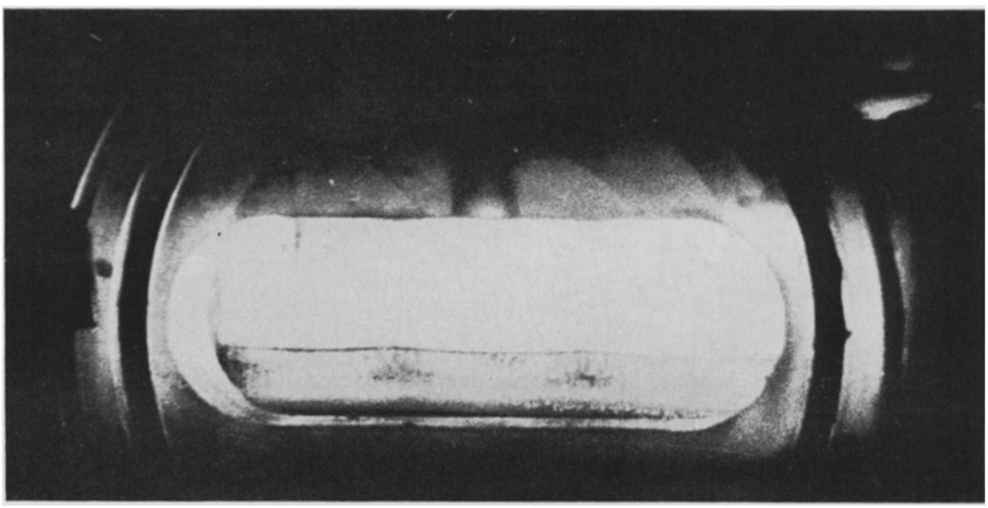

FIG. 12. Photograph of ice surface corresponding to interferogram in Fig. 11.

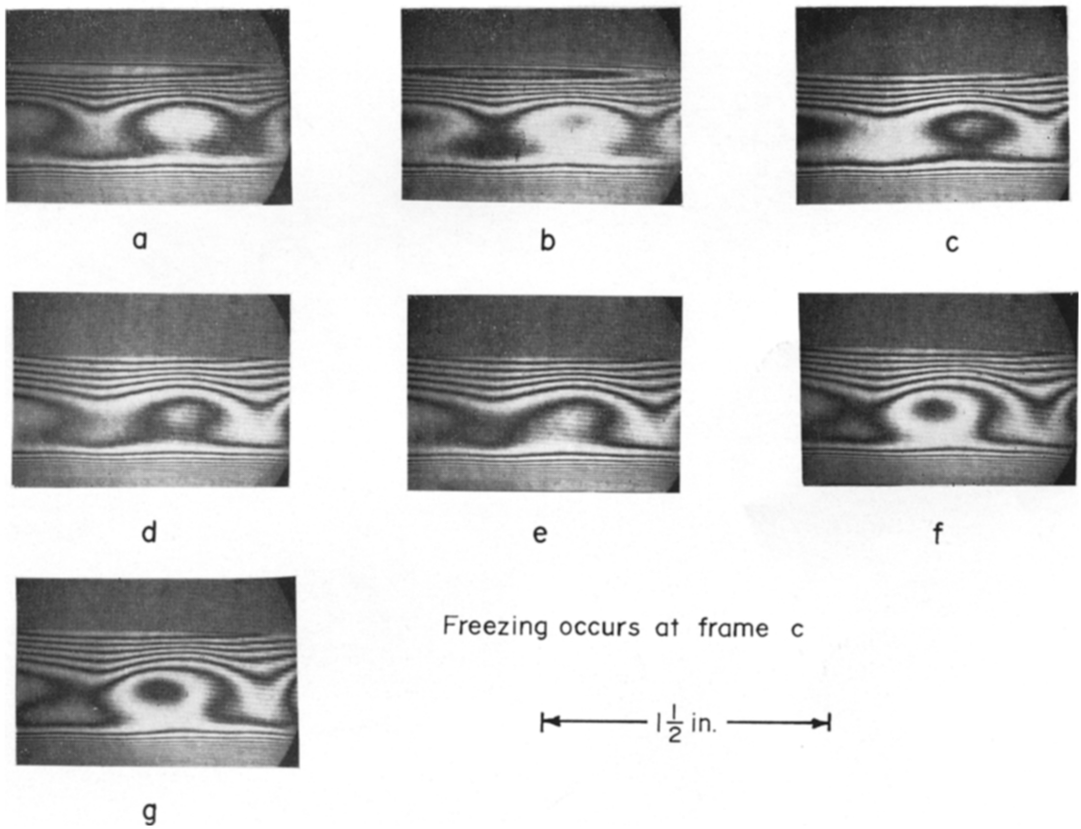

FIG. 13. A sequence of interferograms showing freezing at the top.

thermocouple value. Other such tests show equally good agreement.

From the slope of the temperature distribution in Fig. 4, it appears that the convective region extends well beyond the 4°C isotherm—to about 6°C. It may even extend to higher temperatures if measurements are made on the interferogram along a vertical line passing through the trough. Also it is worth noting that the ratio of the cell spacing to cell height is about 2. This agrees with theory [6, 8]. It should be noted that for temperatures above 5°C, each fringe shift corresponds to about $\frac{1}{4}$°C: thus giving excellent resolution. When necessary, one can easily measure to $\frac{1}{10}$ of a fringe shift. Even in the lower temperature range the resolution is adequate—being least accurate at 0·03°C. However the 0·03°C band provides a reference temperature.

Next the circulation pattern is determined in conjunction with the interferograms. To do this a solid particle of dye (DuPont Victoria Green, 912-73-4P, small crystal) is introduced in the top of the test section. This particle settles to the bottom of the test section and dissolves slowly yielding a dark green color. The dye filaments are carried along by convection currents—thus, forming streak lines. Pictures of the interferogram showing the streak lines are taken at time intervals of about 2·5 s. Figure 5a shows the particle of dye at the bottom and the streak line proceeding along the bottom to the left and upwards. It is noted that the dye particle did not drop straight down. It veered to the right and passed downwards through the trough. Figure 5b, a later picture, shows the dye filament continuing upwards and turning to the right. Figure 5c shows the filament clearly turning downward toward the trough, and Fig. 5d shows the filament almost completing the cycle—returning to its initial position. To clarify these, Fig. 6 is a drawing made by tracing the filaments from the four photographs. The circulation pattern is as follows: The fluid of high density moves down the trough then along the bottom (being cooled) up the center of the peak (being warmed), and finally returns back down the trough. The 4°C isotherm is shown in Fig. 5a. The upward motion of the convection currents exceeds the 4°C isotherm; thus the Bénard cells extend a significant distance into the stable region. This corroborates the conclusion drawn from temperature profiles such as shown in Fig. 4. When the dye particle settles

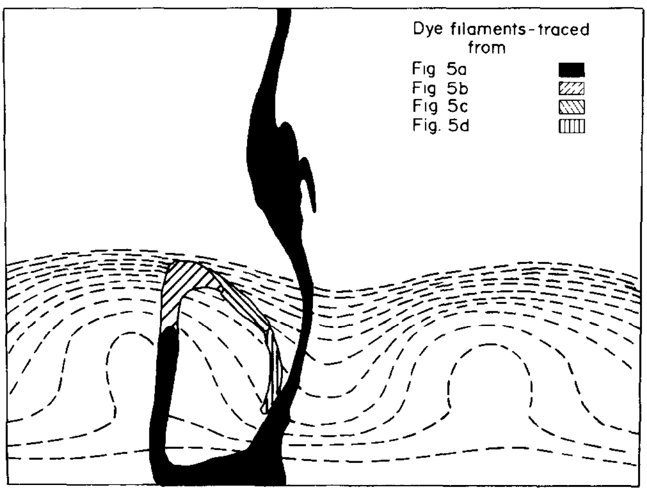

FIG. 6. Tracing of dye filament shown in Fig. 5.

at the center of the trough (as it did in one of the tests), the streak lines go in both directions along the bottom and then upwards along the centerlines of the respective peaks.

By continuously cooling the bottom plate and allowing the top plate temperature to adjust, the 4°C isotherm moves upwards. In Fig. 7, the cells are near the bottom of the test section and relatively close together. The ratio of cell spacing to cell height is about 2. At some time later, the interferogram will look like that shown in Fig. 8. Since the cell height has increased, so has the cell spacing—maintaining the same ratio. The convection pattern (see Fig. 8) can be deduced as follows: The fluid flows down the central half of each trough—along the bottom toward the center of the interferogram—vertically upwards—loops back and returns down the troughs again. The fluid of each of the cells does not mix with the fluid of neighboring cells. Figure 9 is an interferogram from another test for approximately the same conditions. Here the fluid moves downwards in the center—splits with part going right and part going left—and then moves upwards along the center line of their respective peaks. Thus the convection pattern and temperature distribution can be obtained directly from the interferograms.

One point needs clarification. With the present experimental set-up, the roll cells align themselves with their geometric axes parallel to the light beam. This is not likely due to the horizontal dimensions of the test section (2·5 × 2·75 in.). It is probably due to different methods used to insulate the sides of the test section as compared to the front and back of the test section. The guard regions (sides of the test section) are made of $\frac{1}{16}$ in. thick plexiglas partitions that are in direct contact with the upper and lower brass plates (see Fig. 1). The temperatures at the top and botton of the partitions are equal to the temperatures of the upper and lower brass plates respectively and there is a heat flux into the partitions (conduction) from the brass plates. On the other hand, the front and back windows of the test section are thermally floating from the upper and lower brass plates—since they are separated from the plates by rubber gaskets $\frac{1}{8}$ in. thick. An isotherm pattern with troughs and peaks is more readily established at the front and rear windows than along the sides. Thus the rolls align themselves with their geometric axes perpendicular to the front and back windows.

Freezing

The sequence of interferograms shown in Fig. 10 depicts what occurs during the early stages of freezing. Pictures (a)–(g) indicate how the cells develop with time as the 4°C isotherm moves upwards due to cooling of the bottom plate. As shown previously, the warm fluid moves downward in the trough and laterally across the bottom, being cooled (its density decreasing) and then vertically upwards under the peaks. The water in the vicinity of the bottom plate is supercooled. When ice forms, one would not expect a uniform layer; but a layer that is lower under the troughs (warmer water) and higher under the peaks (colder water). Pictures (h)–(p) of Fig. 10 verify this. These pictures were taken at about one second time intervals starting at the instant freezing begins. The ice that first forms is the dentritic type, and it rapidly extends throughout the metastable supercooled region from the bottom plate to approximately the 0°C isotherm. The last picture (q), which was taken about 15 s after picture (p), shows the cell strength has been reduced. A possible explanation of this is as follow: At the base of the trough (of the interferogram), there is a relatively high temperature gradient and relatively little convective heat transfer. Thus the heat transfer at the base of the trough is to a large extent conduction. When ice forms it gives off heat (latent heat of fusion). Initially this heat warms the supercooled fluid (raising the temperature to 0°C) and warms the brass plate (as seen from the thermocouple readings on the digital voltmeter). After the layer of ice forms, the heat conducted from the water to the brass plate (which has begun to cool down again) is reduced. With a reduction in heat transfer to the plate, the isotherms which form the trough must recede from the ice layer to reduce the temperature gradient. Reducing the temperature gradient, reduces the intensity of the Bénard cells. As freezing continues, the ice surface becomes smoother; and the Bénard cells will still persist if the critical Rayleigh number is exceeded. Figure 11 is an interferogram several minutes later when the ice layer has become quite thick. In this picture, the ice surface appears flat even though the convection currents still persist. When the test section was back-lighted with diffuse light and a picture was taken of the interface (see Fig. 12), small bumps are seen on the ice interface which correspond to the peaks in the interferogram: thus convection currents still affect the water–ice interface. The ice shows unexplained dark areas under the bumps which may be due to the ice structure.

Freezing from the top resulted in a water–ice interface that was flat except for the small dendritic irregularities. The convection currents are primarily confined to a region where the temperature is 4°C or greater. These currents penetrate somewhat beyond the 4°C isotherm but do not reach the 0°C isotherm. A sequence of interferograms showing the freezing from the top is seen in Fig. 13. The dendrites extend through the metastable, supercooled region to the 0°C isotherm. As freezing continues, the surface becomes very smooth. No bumps on the water–ice interface were detected.

The Rayleigh number is defined as

$$R = \frac{g\alpha\Delta T d^3}{K\nu} \qquad (2)$$

where g is gravity,

- α is the coefficient of volume expansion,
- d is the depth of the unstable layer,
- K is the thermometric conductivity,
- ν is the kinematic viscosity,
- ΔT is the temperature difference between top and bottom of unstable layer.

The critical Rayleigh number is the value at which the instability first appears. Cells of small depth, just after the onset of instability, are used to determine the critical Rayleigh number. The height of the peak and trough of the 4°C isotherm are averaged to determine the depth of the unstable layer (d). 4°C is taken as the temperature difference (ΔT), and the coefficient of volume expansion (α) was assumed to be half the value at 0°C (average of the 0°C and 4°C values). The computed critical Rayleigh number is 480. This is quite different from the value of 1100 predicted from linearized theory assuming a Boussinesq fluid where one surface is rigid and the other surface is free. Gribov and Gurevich [8] have computed the critical Rayleigh number for an unstable layer when the convection currents which arise in the layer are free to move into the bordering stable regions of the same fluid. They predict a critical Rayleigh number of 106 if the convection currents are free to move into stable regions from both boundaries, and 370 if the convection currents are free to move through only on boundary. The latter condition applies to this experiment. All that can be concluded from the measured critical Rayleigh number is that it appears to be well below 1100 and there is some theoretical justification for this low value.

CONCLUSIONS

(1) The interferometer is an excellent instrument to study two-dimensional Bénard cells. From the interferograms both the temperature distribution in the liquid and the convection pattern are obtained.

(2) When convection currents are in contact with the water–ice interface, the interface is not flat. In the present experiment this occurs when the critical Rayleigh number is exceeded and freezing is from below.

(3) Frequently the lowest critical Rayleigh number is stated to be 1100. For atmospheric and terrestial problems this may not be the case. The critical Rayleigh number in this study is about 480.

(4) It appears that ice, in the form of dentrites, suddenly occurs in the metastable, supercooled

water and extends to the 0°C isotherm. In crystal growth, one of the most important parameters is the interface temperature [9]. An interferometer may be of value in determining this temperature.

(5) The fluctuations in the stable region of the fluid are directly related to the motion in the unstable region. The inertia of the moving fluid in the unstable region carries the fluid beyond the 4°C isotherm and into the stable region.

(6) There is coupling between the ice formed (in the unstable layer) and the convection currents. The original shape of the water–ice interface that appears in the metastable region is controlled by the convection currents. Further growth of ice reduces the intensity of the convection currents, and in turn the Rayleigh number. If freezing occurs when the Rayleigh number is very close to its critical value, the ice layer may reduce it below critical (occurred in one of the tests) and the fluid in contact with the ice becomes stable.

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LES EFFETS DES COURANTS DE CONVECTION THERMIQUE SUR LA FORMATION DE GLACE

Résumé—Un interféromètre Mach–Zender a été utilisé pour étudier le rôle des courants de convection dans la formation de glace. Avec ce montage expérimental, on obtient les isothermes dans le liquide à partir desquels les figures de convection sont déduites. Le nombre critique de Rayleigh correspondant à la glaciation est voisin de 480; quand ce nombre est dépassé, il y a couplage entre l'interface eau–glace et les courants de convection thermique. Les dendrites qui se forment initialement dans la portion d'eau sous-refroidie métastable s'étendent à l'isotherme 0°C.

AUSWIRKUNGEN THERMISCHER KONVEKTIONSSTRÖMUNGEN AUF DIE BILDUNG VON EIS

Zusammenfassung—In einem Mach-Zehnder-Interferometer wurde untersucht, welche Rolle Konvektionsströmungen bei der Bildung von Eis spielen. Mit dieser Versuchsapparatur können die Isothermen in der Flüssigkeit sichtbar gemacht werden und daraus die Konvektionsstromlinien abgeleitet werden. Die kritische Rayleighzahl für das Gefrieren von unten liegt bei 480; wenn diese Zahl überschritten wird, kommt es zu einem Wechselspiel zwischen der Wasser-Eis-Grenze und den thermischen Konvektionsströmungen. Die Dendriten, die sich anfangs im metastabilen, unterkühlten Wasserbereich bilden, dehnen sich bis zur 0°C-Isotherme aus.

ВЛИЯНИЕ ПОТОКОВ ТЕПЛОВОЙ КОНВЕКЦИИ НА ОБРАЗОВАНИЕ ЛЬДА

Аннотация—Для исследования роли конвективных потоков в формировании льда использовался интерферометр Цендера-Маха. На этой эксперементальной установке получены изотермы для жидкости, из которых можно определить режим конвекции. Критическое число Релея при замораживании снизу равно 480. При большем числе Релея имеет место связь между поверхностью раздела вода-лед и конвективными потоками. Дендриты, первоначально образующиеся в метастабильном переохлажденном участке воды, достигают области изотермы 0°C.